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Robust Detection and Accommodation of Incipient Component and Actuator Faults in Nonlinear Distributed Processes

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A class of nonlinear distributed processes with component and actuator faults is presented. An adaptive detection observer with a time varying threshold is proposed that provides additional robustness with respect to false declarations of faults and minimizes the fault detection time. Additionally, an adaptive diagnostic observer is proposed that is subsequently utilized in an automated control reconfiguration scheme that accommodates the component and actuator faults. An integrated optimal actuator location and fault accommodation scheme is provided in which the actuator locations are chosen in order to provide additional robustness with respect to actuator and component faults. Simulation studies of the Kuramoto-Sivashinsky nonlinear partial differential equation are included to demonstrate the proposed fault detection and accommodation scheme.
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Introduction

Actuator, sensor and component failures have too often plagued chemical processes, often leading to deteriorating product quality and potentially dangerous process operation, such as runaway conditions. Motivated by the importance of the aforementioned failures, the issue of fault tolerant and fault accommodating controller design has been an active research topic in the chemical engineering community for open-loop stable and open-loop unstable processes.[1–6] However, while there has been extensive research from the control community on fault detection and diagnosis of finite dimensional systems using model-based robust and adaptive control techniques, see for example the books[7–9] and references therein, little work can be found for similar treatment of infinite dimensional systems.

Previous work on spatially distributed processes has up until recently focused on the design of nonlinear controllers and robust controllers that are specifically tailored to circumvent the computational requirements associated to the infinite dimensional nature of the mathematical description.[10–15] A further complexity of such processes lies in the spatial dependence of the concepts of controllability and observability which prompted a number of researchers to address the important topic of actuator and sensor placement.[16–19]

To address the issue of fault tolerance and using model-based methods along with an adaptive detection observer a fault scheme was first presented in[20,21] for different types of component and actuator faults in general infinite dimensional systems. An adaptive scheme along with robust modifications allowed for the online diagnosis of the component fault parameters. Closer to this work, the concept of a time-varying threshold that minimized the fault detection time was utilized in.[22] Actuator faults with fault tolerant controller design were considered in.[23–27] The concept of actuator/sensor placement

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for minimizing fault detection delay time or improved fault tolerability was first examined in[28] and more recently in.[25]

In this work, we consider a class of infinite dimensional systems that can be decomposed into a finite dimensional slow subsystem and an infinite dimensional fast subsystem. By utilizing a Beard-Jones detection filter for the slow subsystem along with a time-varying threshold similar to the one presented in,[22] the component fault can be detected and the detection time can be reduced over the case of using a constant threshold. Using an adaptive diagnostic observer, the component and actuator faults are accommodated via the use of a control reconfiguration that utilizes online estimates of the fault parameters. Integrated into the adaptive diagnostic and accommodation policy is an actuator optimization scheme that proposes to choose the actuator locations so that the accommodated system would present some robustness with respect to the worst case component fault.

The article is structured as follows. The mathematical framework necessary for examining the class of partial differential equations, is presented initially. The proposed detection observer, i.e., the filter that will be used to detect possible faults in the system, and the diagnostic observer which along with the on-line approximator attempts to learn the system fault are then presented. By utilizing a robust modification in the learning rule of the on-line approximator, the detection and diagnostic observers are combined into a single detection/diagnostic observer and the fault accommodating controller are also presented in the subsequent section. Additionally, an actuator placement scheme is also presented in the following section that allows one to incorporate robustness with respect to actuator locations. Subsequently, extensive numerical studies are presented to demonstrate the proposed scheme. Finally, conclusions with future work that is currently being considered by the authors are summarized.

We use the standard notation $\langle \phi, \psi \rangle$ for the L_2 inner product of two functions $\phi(\xi)$ and $\psi(\xi)$ and for the transpose of a column vector, we use $x^T(t)$.

Mathematical formulation

Consider a distributed process, often described by nonlinear partial differential equations, represented by the following evolution equation in an appropriate abstract space $\mathcal{H}$

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{N}(x(t)) + \mathcal{B}u(t) \tag{1}$$

where $\mathcal{A}$ represents a linear spatial operator, the locally Lipschitz term $\mathcal{N}$ denotes the nonlinear dynamics, and $\mathcal{B}$ denotes the control input operator. Alternatively, the term $\mathcal{N}(x(t))$ may be thought of as the term representing unmodeled dynamics. The aforementioned is considered to represent the *healthy system* dynamics.

Both component and actuator faults may be considered in this framework and which result in the modified dynamics of the healthy system (1)

$$\begin{aligned}\dot{x}(t) = \mathcal{A}x(t) + \mathcal{N}(x(t)) + \mathcal{B}u(t) + \beta_a(t - T_a)\mathcal{B}f_a(t)\\ + \beta_c(t - T_c)\Xi(x(t))\end{aligned} \tag{2}$$

where $\beta_a(t - T_a)$ denotes the time profile of the actuator fault, and T_a denotes the time occurrence of the actuator fault. Similarly, $\beta_c(t - T_c)$ and T_c denote the time profile and time occurrence of the component fault, respectively. The *time profiles* of the actuator and component faults may represent both *abrupt* and *incipient* profiles, and are given by

$$\beta_i(t - T_i) = \begin{cases} 0 & \text{if } t < T_i \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

for abrupt fault profiles, and by

$$\beta_i(t - T_i) = \begin{cases} 0 & \text{if } t < T_i \\ 1 - e^{-\lambda(t - T_i)} & \text{otherwise} \end{cases} \tag{4}$$

for incipient fault profiles. Alternatively, one may consider the abrupt fault profiles as a special case of incipient faults with a fault profile rate $\lambda \to \infty$. Additive actuator faults are represented by f_a, and the component fault is represented by the term $\Xi(x(t))$.

In this work, we focus our attention to distributed processes that exhibit strong diffusive phenomena, and for which the spatial operator $\mathcal{A}$ is a strongly elliptic spatial operator.[29] A finite-dimensional approximation of the evolution system (2) realized through the slow eigenmodes of the differential operator $\mathcal{A}$ can be derived, as it naturally emerges from the time-scale decomposition of the operator's eigenspectrum, obtained from solution of the eigenvalue-eigenfunction problem $\mathcal{A}\phi = \mu\phi$.[30–32] To further the subsequent exposition we assume that the eigenvalues are ordered in decreasing value

$$\mu_{i+1} \leq \mu_i, \quad \forall i = 1, ..., \infty, \quad \text{and} \quad \left|\frac{\Re\{\mu_1\}}{\Re\{\mu_n\}}\right| \approx 1,$$

$$\left|\frac{\Re\{\mu_n\}}{\Re\{\mu_{n+1}\}}\right| = \varepsilon \ll 1$$

In summary, we consider the decomposition $H = H_s \oplus H_f$, in which H_s denotes the finite dimensional space spanned by the unstable/slow part of $\mathcal{A}$'s eigenspectrum H_s = span $\{\varphi_1,\ldots,\varphi_n\}$ and H_f = span$\{\varphi_{n+1}, \varphi_{n+2},\ldots\}$ is the infinite-dimensional complement one spanned by the stable/fast eigenfunctions. Under the previous decomposition, we define the orthogonal projection operators $\mathcal{P}_s : H \mapsto H_s$ and $\mathcal{P}_f : H \mapsto H_f$, that yield the following decomposition for the state

$$x = \mathcal{P}_s x + \mathcal{P}_f x = x_s + x_f$$

Application of the projection operators $\mathcal{P}_s$ and $\mathcal{P}_f$ to the aforementioned system yields the following equivalent form of the process dynamics

$$\begin{aligned}\frac{dx_s}{dt} &= \mathcal{A}_s x_s + \mathcal{B}_s(u(t) + \beta_a(t - T_a)f_a(t)) + \mathcal{N}_s(x_s + x_f)\\ &\quad + \beta_c(t - T_c)\Xi_s(x_s + x_f)\\ \frac{dx_f}{dt} &= \mathcal{A}_f x_f + \mathcal{B}_f(u(t) + \beta_a(t - T_a)f_a(t)) + \mathcal{N}_f(x_s + x_f)\\ &\quad + \beta_c(t - T_c)\Xi_f(x_s + x_f)\\ x_s(0) &= \mathcal{P}_s x(0), \quad x_f(0) = \mathcal{P}_f x(0)\end{aligned} \tag{5}$$

$\mathcal{A}_s$ is an n-dimensional matrix with diagonal structure, $\mathcal{A}_s = \text{diag}\{\mu_i\}$, and the unbounded operator $\mathcal{A}_f$ is an infinitesimal generator of an exponentially stable C_0 semigroup.[33] Furthermore, under standard assumptions, one can show that

$$\mathcal{N}_s(x) = \mathcal{P}_s\mathcal{N}(x) = \mathcal{N}_s(x_s + x_f),$$
$$\mathcal{N}_f(x) = \mathcal{P}_f\mathcal{N}(x) = \mathcal{N}_f(x_s + x_f) \tag{6}$$

are also Lipschitz continuous vector functions.

Before we proceed with the design of a robust adaptive detection observer, we provide bounds of the state in the fault-free case; such bounds will allow for the design of time-varying thresholds used in the fault detection stage via the use of residual signals.

Under the assumption that the control action and the onset of faults are sufficiently slow, and using singular perturbation arguments, we may neglect the fast and stable infinite dimensional x_f subsystem to obtain the n-dimensional slow system

$$\dot{\bar{x}}_s(t) = \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s(u(t) + \beta_a(t - T_a)f_a(t)) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t - T_c)\Xi_s(\bar{x}_s(t)) \tag{7}$$

that provides an $O(\varepsilon)$ approximation of (5) after a short relaxation period.[34–37]

Problem statement: The primary objective is to design a monitoring filter that would provide information on the time occurrence of the fault and attempt to diagnose the nature of the component fault. The secondary objective is to then provide a control reconfiguration policy based on the estimates of the fault, and which would be able to accommodate the faults.

Main Results: Robust Detection, Diagnosis and Accommodation

The main results for component faults are presented in this section. While the system (2) also includes actuator faults, the treatment here considers only component faults for ease of exposition. These can easily be extended to include actuator faults, but since these include arguments from earlier work on actuator fault detection and accommodation for general infinite dimensional systems, the interested reader may refer to[22,23] for an in-depth exposition.

Detection observer

The proposed detection observer is then designed around the slow subsystem given in Eq. (7) earlier. Toward this end, we propose a detection observer for the system having only component faults and unmodeled dynamics

$$\dot{\bar{x}}_s(t) = \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s u(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t - T_c)\Xi_s(\bar{x}_s(t)) \tag{8}$$

The detection observer takes the form

$$\frac{d}{dt}\hat{x}_s(t) = \mathcal{A}\hat{x}_s(t) - \mathcal{L}_s(\hat{x}_s(t) - \bar{x}_s(t)) + \mathcal{B}_s u(t),$$
$$\hat{x}_s(0) = \hat{x}_0 \neq \hat{x}_s(0). \tag{9}$$

This is an exact copy of the nominal plant with a corrective term $\mathcal{L}_s(\hat{x}_s(t) - \bar{x}_s(t))$, where $\mathcal{L}_s$ is an appropriately chosen filter gain. The state observation error will serve as the residual signal used in fault detection/diagnosis. The error $e_s(t) = \bar{x}_s - \hat{x}_s(t)$ is governed by

$$\dot{e}_s(t) = (\mathcal{A}_s - \mathcal{L}_s)e_s(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t - T_c)\Xi_s(\bar{x}_s(t)) \tag{10}$$

Even when one sets $\hat{x}_s(0) = \bar{x}_s(0)$, the state estimation error will be nonzero prior to the fault occurrence ($t < T_c$), due to the presence of the unmodeled/nonlinear dynamics term $\mathcal{N}_s(\bar{x}_s(t), 0)$. To avoid false declaration of faults, one considers a threshold which, when exceeded, designates the occurrence of the component fault in the system. To find such a threshold, one considers the above state observation error in the absence of faults, *but* with the inclusion of the unmodeled/nonlinear dynamics

$$\dot{e}_s(t) = (\mathcal{A}_s - \mathcal{L}_s)e_s(t) + \mathcal{N}_s(\bar{x}_s(t)). \tag{11}$$

Its solution is given by

$$e_s(t) = e^{(\mathcal{A}_s - \mathcal{L}_s)t}e_s(0) + \int_0^t e^{(\mathcal{A}_s - \mathcal{L}_s)(t-\tau)}\mathcal{N}_s(\bar{x}_s(\tau))d\tau. \tag{12}$$

The threshold is then given by the norm bound

$$\begin{aligned}\|e_s(t)\| &= \left\| e^{(\mathcal{A}_s - \mathcal{L}_s)t}e_s(0) + \int_0^t e^{(\mathcal{A}_s - \mathcal{L}_s)(t-\tau)}\mathcal{N}_s(\bar{x}_s(\tau))d\tau \right\| \\ &\leq \left\| e^{(\mathcal{A}_s - \mathcal{L}_s)t}e_s(0) \right\| + \left\| \int_0^t e^{(\mathcal{A}_s - \mathcal{L}_s)(t-\tau)}\mathcal{N}_s(\bar{x}_s(\tau))d\tau \right\| \\ &\leq e^{-\alpha t}\|e_s(0)\| + \int_0^t e^{-\alpha(t-\tau)}\|\mathcal{N}_s(\bar{x}_s(\tau))\|d\tau, \end{aligned} \tag{13}$$

where α is the spectrum bound of the matrix $(\mathcal{A}_s - \mathcal{L}_s)$. Using the *a priori* ($L_\infty$) bound on the unmodeled/nonlinear dynamics $\|\mathcal{N}_s(\bar{x}_s(\tau))\| \leq \mathcal{N}_0$ for all $\bar{x}_s$ in a compact set in $\mathbb{R}^m$, we obtain the *time varying threshold*

$$r_0(t) = e^{-\alpha t}\varepsilon_{s0} + \frac{1 - e^{-\alpha t}}{\alpha}\mathcal{N}_0, \quad t \geq 0 \tag{14}$$

where ε_{s0} is the bound in $\|e_s(0)\| \leq \varepsilon_{s0}$. We now proceed with the following result on robust fault detection scheme, which assumes an observability result for the fault via the appropriate decoupling of unmodeled dynamics and fault dynamics. The novelty of the standard detection observer is that in this case the residual threshold is time varying, thereby minimizing the detection time, i.e., the time elapsed from a fault occurrence till its declaration by the supervisor.

Lemma 1 (Detection observer). Consider the approximate finite dimensional slow subsystem with incipient component faults

$$\dot{\bar{x}}_s(t) = \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s u(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t - T_c)\Xi_s(\bar{x}_s(t))$$
$$\bar{x}_s(0) = \bar{x}_{s0}.$$

It is assumed that the fault function $\Xi_s(\bar{x}_s(t))$ is such that it decouples from the unmodeled/nonlinear dynamics, in the sense that the the unmodeled dynamics cannot mask the fault dynamics. Then, a fault is declared when the residual signal, given by the norm of the state observation error $e_s = \bar{x}_s(t) - \widehat{x}_s(t)$ and governed by

$$\dot{e}_s(t) = (\mathcal{A}_s - \mathcal{L}_s)e_s(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t - T_c)\Xi_s(\bar{x}_s(t))$$
$$e_s(0) = e_{s0} \neq 0$$

exceeds the time varying threshold $r_0(t)$ given in (14). The norm of the state error is bounded by $r_0(t)$ for all $t \leq T$. Fault is declared at the detection time t_d

$$t_d = \inf(t > 0 : \|e_s(t)\| \geq r_0(t))$$

The proof of Lemma 1 can be found in Appendix A.

Remark 1: When the initial condition $\bar{x}_s(0)$ is known, then one may initialize $\bar{x}_s(0)$ to further tighten the time varying threshold of the residual signal in (14), which is now given by

$$r_0(t) = \frac{1 - e^{-\alpha t}}{\alpha}\mathcal{N}_0, \quad t \geq 0$$

This tighter bound of the residual signal will help minimize the detection time t_d.

Adaptive Diagnostic Observer

Once a fault is declared, one may subsequently activate a diagnostic observer in order to diagnose/isolate the component faults. It is assumed that the component fault term admits the following parametrization

$$\Xi_s(\bar{x}_s(t)) = \Theta g(\bar{x}_s(t)) \tag{15}$$

where the $m \times m$ constant matrix Θ is assumed unknown and desired to be identified, and the vector field $g(\bar{x}_s)$ is assumed known. The adaptive diagnostic scheme consists of a diagnostic observer and a parameter learning scheme given by

$$\dot{\widehat{x}}_d(t) = \mathcal{A}_s\widehat{x}_d(t) - \mathcal{L}_d(\widehat{x}_d(t) - \bar{x}_s(t)) + \mathcal{B}_s u(t) + \widehat{\Theta}(t)g(\bar{x}_s(t))$$
$$\dot{\widehat{\Theta}}(t) = \mathcal{P}\{\Gamma g(\bar{x}_s(t))e_d^T(t)\Pi\} \tag{16}$$

where $\widehat{x}_d(t) \in \mathbb{R}^m$ is the *estimated state vector*, $\widehat{\Theta}(t)g(\bar{x}_s(t))$ is the adaptive fault approximator model with $\widehat{\Theta}(t) \in \mathbb{R}^{m\times m}$ being a matrix of (adaptively) *adjustable* parameters, Γ is a constant $m \times m$ matrix of adaptive gains,[38] $e_d(t) \stackrel{\Delta}{=} \bar{x}_s(t) - \widehat{x}_d(t)$ is the diagnostic state estimation error, and $\mathcal{L}_d$ is a constant $m \times m$ observer gain matrix that satisfies the following Lyapunov equation

$$(\mathcal{A}_s - \mathcal{L}_d)\Pi + \Pi(\mathcal{A}_s - \mathcal{L}_d)^T = -Q \tag{17}$$

with $\Pi = \Pi^T > 0$ and $Q > 0$. It is found in the similar way as in the detection observer, and one may in fact use the same filter gain for both observers. Note that the diagnostic state estimation error $e_d(t) = \bar{x}_s(t) - \widehat{x}_d(t)$ differs from the detection state estimation error $e_s(t)$. The difference between (9) and (16) is the additional learning term $\widehat{\Theta}(t)g(\bar{x}_s(t))$ in (16). The projection operator $\mathcal{P}[\cdot]$ constrains the parameter $\widehat{\Theta}$ to an *a priori* selected compact convex region $\mathcal{M}$ of the parameter space $\mathbb{R}^{m\times m}$. When $\widehat{\Theta}(t)$ is not in the set $\mathcal{M}$, the adaptation is terminated. This is described in detail in,[39] and when applied to the case under consideration, is given by

$$\dot{\widehat{\Theta}}(t) = \mathcal{P}\{\Gamma g(x_s(t))e_d^T(t)\Pi\}$$
$$= \begin{cases} \Gamma g(x_s)e_d^T(t)\Pi & \text{if } \widehat{\Theta} \in \mathcal{M}^0 \text{ or if } \widehat{\Theta} \in \partial\mathcal{M} \text{ and} \\ & \quad \widehat{\Theta} \text{ tends to move towards } \mathcal{M} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Before we consider the stability and convergence properties of the above adaptive diagnostic observer, we provide the definition of *Persistence of Excitation* as it guarantees parameter convergence.

Definition 1 (Persistence of Excitation) The nonlinear system is said to be persistently excited, if there exists T_0, δ_0 and ε_0 such that for each $\Theta^* \in \mathbb{R}^{m\times m}$ with unit norm, and each $t > T_c$ sufficiently large, there exists a $\widetilde{t} \in [t, t + T_0]$ such that

$$\left| \int_{\widetilde{t}}^{\widetilde{t}+\delta_0} \Theta^* g(\bar{x}_s(\tau)d\tau) \right| \geq \varepsilon_0$$

The following lemma examines the stability properties of the aforementioned diagnostic observer.

Lemma 2 (Diagnostic Observer). Consider the post-fault finite dimensional subsystem

$$\dot{\bar{x}}_s(t) = \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s u(t) + \mathcal{N}_s(\bar{x}_s(t))$$
$$+\beta_c(t - T_c)\Xi_s(\bar{x}_s(t)) \tag{19}$$
$$\bar{x}_s(T_c) = \bar{x}_s T_c \qquad t \geq (T_c)$$

Once the fault is declared via the adaptive detection observer, then the following diagnostic observer plus online approximator

$$\dot{\widehat{x}}_d(t) = \mathcal{A}_s\widehat{x}_d(t) - \mathcal{L}_d(\widehat{x}_d(t) - \widehat{x}_s(t))$$
$$+\mathcal{B}_s u(t) + \widehat{\Theta}(t)g(\widehat{x}_s(t))$$
$$\widehat{x}_d(t_d) = \widehat{x}_{d0} \qquad t \geq t_d > T_c \tag{20}$$
$$\dot{\widehat{\Theta}}(t) = \mathcal{P}\{\Gamma g(x_s(t))e_d^T(t)\Pi\}$$
$$\widehat{\Theta}(t_d) = \widehat{\Theta}_d$$

guarantees that all signals are bounded, and in the special case of zero unmodeled/nonlinear dynamics, one obtains convergence of the state diagnostic error to zero. Additionally, when the system is assumed to be persistently exciting as in Definition 1, then convergence of $\widehat{\Theta}(t)$ to Θ is guaranteed (parameter convergence).

The proof of Lemma 2 can be found in Appendix B.

One may combine the detection and adaptive diagnostic observer, by ensuring that during the detection stage-with no fault present-no adaptation takes place. This is to avoid incorporating a contribution to the state error due to the a falsely estimated component fault

$$\dot{e}(t) = \mathcal{A}_{so}e(t) + \mathcal{N}_s(\bar{x}_s(t)) - \widehat{\Theta}(t)g(\bar{x}_s(t))$$

which leads to

$$\begin{aligned} r(t) &= \| e_s(t) \| \\ &\leq e^{-\alpha t} | \| e_s(t) \| + \int_0^t e^{-\alpha(t-\tau)} \| \mathcal{N}_s(\bar{x}_s(\tau)) \| \, d\tau \\ &\quad + \int_0^t e^{-\alpha(t-\tau)} \widehat{\Theta}(\tau) \, \| g(\bar{x}_s(\tau)) \| \, d\tau \\ &\leq r_0(t) + \int_0^t e^{-\alpha(t-\tau)} \widehat{\Theta}(\tau) \, \| g(\bar{x}_s(\tau)) \| \, d\tau. \end{aligned}$$

Hence, prior to the fault occurrence ($t \leq T_c$), one may have $r(t) \geq r_0(t)$ if no measures are taken to avoid false declarations of faults.

Integrated adaptive detection and diagnostic observer

By incorporating a robust modification in the adaptation, one may guarantee that no adaptation takes place whenever $r(t) < r_0(t)$. This is done via a *dead-zone projection*.[38]

Lemma 3. Consider the system (7) with the component fault given by (15). The following integrated adaptive detection/diagnostic observer

$$\begin{aligned} \dot{\hat{x}}_d(t) &= \mathcal{A}_s\hat{x}_d(t) - \mathcal{L}_d(\hat{x}_d(t) - \bar{x}_s(t)) + \mathcal{B}_s u(t) \\ &\quad + \widehat{\Theta}(t)g(\bar{x}_s(t)) \\ \hat{x}_d(0) &= \hat{x}_0 \qquad\qquad t \geq 0 \\ \dot{\widehat{\Theta}}(t) &= \mathcal{P}\{\Gamma g(x_s(t))\mathcal{D}[e^T(t)]\Pi\} \\ \widehat{\Theta}(t_d) &= \widehat{\Theta}_d \end{aligned} \tag{21}$$

where, $e(t) \triangleq \bar{x}_s(t) - \hat{x}_d(t)$ and

$$\mathcal{D}[e^T(t)] = \begin{cases} 0 & \text{if} |e(t)| \leq r_0(t) \\ e^T(t) & \text{otherwise} \end{cases}$$

guarantees that

1. prior to the fault, the residual will be less than the threshold, and no adaptation of $\widehat{\Theta}(t)$ will take place.
2. at the onset of the fault, a fault is declared when the residual exceeds the threshold $r_0(t)$.
3. once a fault is declared, the dead-zone modification activates the adaptation of $\widehat{\Theta}(t)$.
4. once adaptation is activated, all signal remain bounded, and in the special case of zero unmodeled/nonlinear dynamics, one obtains convergence of the state diagnostic error to zero, and with the additional assumption of persistence of excitation convergence of $\widehat{\Theta}(t)$ to Θ (parameter convergence).

Fault Accommodation

Once the component fault is declared and the associated diagnostic observer is activated, then one may reconfigure the control signal of the healthy system in order to accommodate the faults. However, this is possible when the term $\mathcal{N}_s(\bar{x}_s(t))$ represents the known nonlinear dynamics of the healthy system (1).

We first consider the case where the component fault is known, but its occurrence is unknown. Once it is declared, one may augment the control signal for the healthy system with a signal that at most can cancel the additional dynamics due to the fault, and at least to include an additional stabilizing component in order to preserve the stability of the closed loop system in the very presence of the component faults. Denote by $u_0(t)$ the control signal for the fault-free system

$$\dot{\bar{x}}_s(t) = \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s u(t) + \mathcal{N}_s(\bar{x}_s(t))$$

and given by

$$u_0(t) = -\mathcal{B}_s^{-1}\mathcal{N}_s(\bar{x}_s(t)) - \mathcal{K}_s\bar{x}_s(t) \tag{22}$$

where the feedback gain $\mathcal{K}_s$ is an appropriately chosen gain that satisfies certain performance and stability criteria for the healthy system. The above control signal yields the following closed loop fault-free slow subsystem

$$\dot{\bar{x}}_s(t) = (\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)\bar{x}_s(t)$$

The aforementioned system describes the (idealized) behavior of the fault-free closed loop subsystem. When the component faults are present in the slow subsystem

$$\begin{aligned} \dot{\bar{x}}_s(t) &= \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s u(t) + \mathcal{N}_s(\bar{x}_s(t)) + \Xi_s(\bar{x}_s(t)) \\ &= \mathcal{A}_s\bar{x}_s(t) + \mathcal{B}_s u(t) + \mathcal{N}_s(\bar{x}_s(t)) + \Theta g(\bar{x}_s(t)) \end{aligned}$$

then the fault accommodating controller

$$u_{accom}(t) = u_0(t) - \mathcal{B}_s^{-1}\Xi_s(\bar{x}_s(t))$$

will cancel the effects of the component fault and yield

$$\dot{\bar{x}}_s(t) = (\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)\bar{x}_s(t)$$

However, the previous fault accommodating controller cannot be implemented as the knowledge of the additional dynamics due to the component fault are not known. In that case, one replaces them by their adaptive estimates

$$u_{accom}(t) = u_0(t) - \mathcal{B}_s^{-1}\widehat{\Theta}(t)g(\bar{x}_s(t)) \tag{23}$$

to arrive at the closed loop system

$$\dot{\bar{x}}_s(t) = (\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)\bar{x}_s(t) - \widetilde{\Theta}(t)g(\bar{x}_s(t)) \tag{24}$$

To examine the stability of the closed loop system, one must simultaneously consider the diagnostic observer and the slow subsystem. The following Lyapunov function is then considered

$$V_{cl} = V + \frac{1}{2}\bar{x}_s^T(t)\Pi_s\bar{x}_s(t)$$

Unlike the stability arguments used in the diagnostic observer, one cannot assume *a priori* the uniform boundedness of the slow subsystem. However, when one assumes that $|g(\bar{x}_s)| \leq k_1|\bar{x}_s|$ and $|\mathcal{N}_s(\bar{x}_s(t))| \leq k_2|\bar{x}_s|$, then

$$\dot{V}_{cl} \leq -\alpha_1|e_s(t)|^2 - \alpha_2 \| \Phi(t) \|_2^2 - \alpha_3|\bar{x}_s(t)|^2 \leq 0$$

for some $\alpha_i > 0$. Application of Barbălat's lemma[38] yields convergence of the slow subsystem state to zero.

Remark 2: It should be noted that the aforementioned fault accommodating controller can be automated, in the sense of including the term $-\mathcal{B}_s^{-1}\widehat{\Theta}(t)g(\bar{x}_s(t))$, and ensuring that no parameter adaptation will take place prior to the fault declaration. This can be done via a deadzone modification in the adaptation law

$$\dot{\widehat{\Theta}}(t) = \mathcal{P}\{\Gamma g(x_s(t))\mathcal{D}[e^T(t)]\Pi\}$$

where

$$\mathcal{D}[e^T(t)] = \begin{cases} 0 & \text{if } |e(t)| \le r_0(t) \\ e^T(t) & \text{otherwise} \end{cases}$$

and which ensures that the term $\widehat{\Theta}(t)$ is zero prior to the fault declaration.

Remark 3: An implicit assumption made in the ability of the system to accommodate the fault via control reconfiguration is that the component fault term $\Xi(x(t))$ will not alter the dimension of the slow/unstable system, i.e., it will not increase the number of unstable modes. A more stringent condition may also be imposed

$$\langle \mathcal{A}x(t) + \mathcal{B}u_0(t) + \mathcal{N}(x_s(t)) + \Xi(x(t)), x(t)\rangle \le 0$$

which states that the controller based on the finite dimensional slow/unstable system will have authority over the closed loop system with component faults.

Actuator location optimization

One may provide an additional degree of freedom of the fault accommodating controller

$$u_{accom}(t) = \underbrace{-\mathcal{B}_s^{-1}\mathcal{N}_s(\bar{x}_s(t)) - \mathcal{K}_s\bar{x}_s(t)}_{u_0(t)} - \mathcal{B}_s^{-1}\widehat{\Theta}(t)g(\bar{x}_s(t)) \quad (25)$$

by parameterizing the control distribution matrix $\mathcal{B}_s$ by the candidate actuator locations. It is assumed that an admissible set of locations exists that renders the finite dimensional subsystem controllable. However, since the controller structure for the fault-free system in (25) requires an invertible control input distribution matrix, then

$$\Psi = \{\psi \in \Omega : \mathcal{B}_s(\psi) \text{ is invertible}\}$$

The location-parameterized accommodating controller then becomes

$$u_{accom}(t;\psi) = \underbrace{-\mathcal{B}_s^{-1}(\psi)\mathcal{N}_s(\bar{x}_s(t)) - \mathcal{K}_s(\psi)\bar{x}_s(t)}_{u_0(t;\psi)} - \mathcal{B}_s^{-1}(\psi)\widehat{\Theta}(t)g(\bar{x}_s(t)).$$

It results in the Ψ-parameterized, fault-accommodated closed loop system

$$\dot{\bar{x}}_s(t) = (\mathcal{A}_s - \mathcal{B}_s(\Psi)\mathcal{K}_s(\Psi))\bar{x}_s(t) - \widetilde{\Theta}(t)g(\bar{x}_s(t)) \quad (26)$$

One may search for actuator locations, constrained in Ψ, such that the effect of the term $\widetilde{\Theta}(t)g(\bar{x}_s(t))$ on the slow subsystem state $\bar{x}_s(t)$ is minimized. However, the adaptive estimate $\widetilde{\Theta}(t)$ is not known. Only an upper bound is known (from the projection modification), and this may be used in the location optimization. In summary, one finds the "worst" value of the term $\widetilde{\Theta}(t)g(\bar{x}_s(t))$ and then optimizes the actuator locations that would minimize the effects of the worst such term on the state of the slow subsystem $\bar{x}_s(t)$. The appropriate measure can be taken as the $\mathcal{H}^\infty$ or $\mathcal{H}^2$ norm of the associated transfer function of (26). This is summarized below: assume that one can find the "worst" value $\widetilde{\Theta}^*$ of $\widetilde{\Theta}(t)$ and $g(x)$ can be locally bounded above and below by linear vector functions of $\underline{G}x \le g(x) \le \bar{G}x$ Then the optimal actuator locations are found via

$$\psi = \arg\min_{\psi\in\Psi}\ \max_{G\in\{\underline{G},\bar{G}\}} \| (sI - (\mathcal{A}_s - \mathcal{B}_s(\psi)\mathcal{K}_s(\psi) - \widetilde{\Theta}^*G))^{-1}\|_\infty$$

and which essentially produces locations that minimize the effects of the "worst" term $\widetilde{\Theta}^* g(\bar{x}_s(t))$ on the state of the slow subsystem. Such a worst term can be found as the one that maximizes its effects on the state of the slow subsystem, and, thus, the optimization is reformulated as a minmax problem

$$\psi = \arg\min_{\psi\in\Psi}\ \max_{\widetilde{\Theta}\in\mathcal{M}, G\in\{\underline{G},\bar{G}\}} \| (sI - (\mathcal{A}_s - \mathcal{B}_s(\psi)\mathcal{K}_s(\psi) - \widetilde{\Theta}G))^{-1}\|_\infty$$

The optimal location problem can also be solved for such linear systems employing the notion of spatial H_2 norm.[17]

Remark 4: The optimal actuator location problem may also be solved employing directly a state-space representation of the system dynamics, through the formulation of a dynamic optimization problem with a finite time horizon.[40,41] This problem formulation will circumvent the need for linear bounding functions of $g(x)$. We note that even though the two optimization problems will be similar, the respective formulations will not be equivalent.

Example and Numerical Results

The proposed methodology was employed to the Kuramoto-Sivashinsky which can adequately describe incipient instabilities arising in a variety of physical and chemical systems including Belouzov Zabotinskii reaction patterns,[42,43] unstable flame fronts,[44] interfacial instabilities between two viscous fluids[45] and falling liquid films.[46]

Specifically, for presentation purposes, the one-dimensional controlled Kuramoto-Sivashinsky equation is considered in the bounded interval $\Omega = [-\pi, \pi]$

$$\frac{\partial U(\xi,t)}{\partial t} + \nu\frac{\partial^4 U(\xi,t)}{\partial \xi^4} + \frac{\partial^2 U(\xi,t)}{\partial \xi^2} + U(\xi,t)\frac{\partial U(\xi,t)}{\partial \xi} = \sum_{i=1}^{m} b_i(\xi)u_i(t) + \beta(t-T_c)\theta\frac{\partial^2 U(\xi,t)}{\partial \xi^2}$$

along with the periodic boundary conditions

$$\frac{\partial^j U}{\partial \xi^j}(-\pi, t) = \frac{\partial^j U}{\partial \xi^j}(\pi, t), \quad j: 0, 1, 2, 3$$

and the initial condition

$$U(\xi, 0) = U_o(\xi).$$

$U(\xi,t)$ denotes the state of the system, $\xi \in [-\pi, \pi]$ is the spatial coordinate, and v denotes the instability parameter. The control signals $u_i \in \mathbb{R}$ describe the temporal components of the external excitation and the functions $b_i(\xi)$ are the actuator distribution functions, describing the spatial influence of the actuating devices. The component fault models an uncertainty in the parameter of the Laplace operator. The previous equation can be placed in the abstract form given by (1) in Appendix C.

For the numerical value of $v = 0.1$, we observe that the eigenvalues obtain the values $\lambda_1 = 0.9$, $\lambda_2 = 2.4$, $\lambda_3 = 0.9$ and $\lambda_4 = -9.6$, resulting in the dimension of the slow/unstable subsystem to be $m = 3$. Certainly, $|\lambda_3/\lambda_4| = 0.0938 \ll 1$ which justifies the chosen time-scale decomposition. Both an abrupt (i.e., $\beta(t - T) = H(t - T)$) and an incipient (i.e., $\beta(t - T) = 1 - e^{-1.2(t-T)}$) profile are considered with the fault occurrence chosen at $T = 0.85$.

Figure 1 depicts the evolution of the residual signal for the system subjected to an abrupt fault occurring at $T = 0.85$. Using the fault accommodation presented earlier, the system detects the fault at $t = 1.465$ when using a time-varying threshold, and at $t = 1.81$, when using a fixed threshold. The effects of fault accommodation, with and without time varying thresholds, is presented in Figure 2. The state norm converges to zero faster when accommodation with a time varying threshold is utilized. A somewhat slower convergence is observed when a fixed threshold is used for fault detection. When no fault accommodation is implemented, the system becomes unstable.

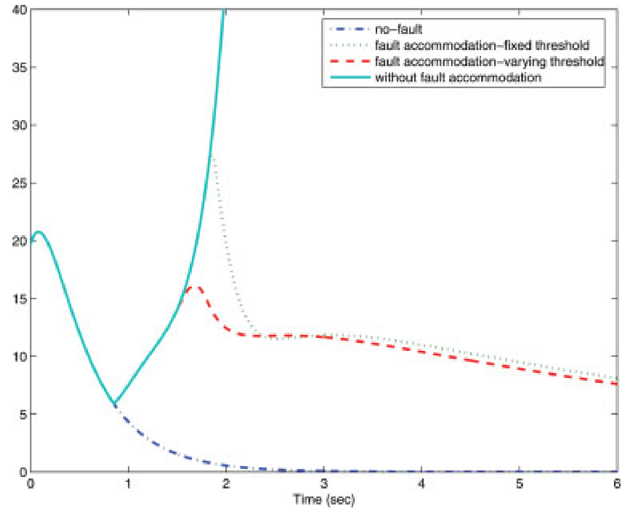

Figure 2. Evolution of state norm using fixed and time-varying thresholds in abrupt fault detection declaration.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Next, an incipient time profile is used and it can be observed that with a time-varying threshold, the fault is detected at $t = 2.615$, thereby producing a detection time of $\Delta t = 1.765$. Figure 3 depicts the evolution of the residual, and Figure 4 depicts the evolution of the state norm of the system with fault accommodation. Additionally, the state norm under the no-fault case (dotted) is given. Finally, the evolution of the term $\theta\beta(t - T)$ along with its online estimate (solid) are presented in Figure 5. One may also observe the value of the time instance of the fault detection to be that of the time that the adaptation is activated. The parameter estimate, with the aid of the projection modification, constrains the estimate $\widehat{\Theta}(t)$ to a bounded value. Additionally,

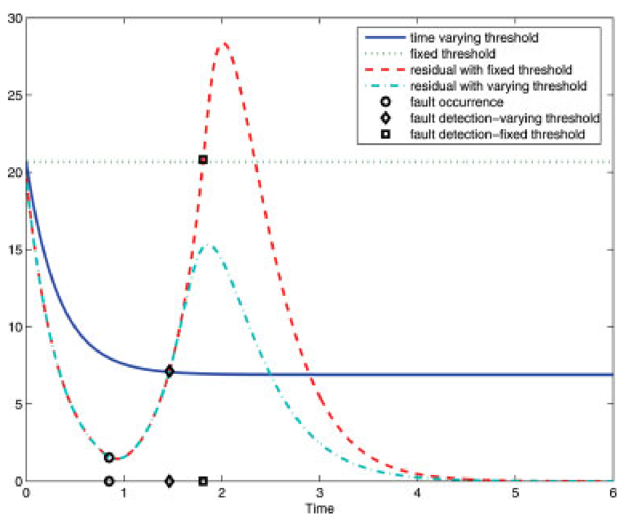

Figure 1. Evolution of residual signals *r*(*t*) using fixed and time-varying thresholds in abrupt fault detection declaration.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

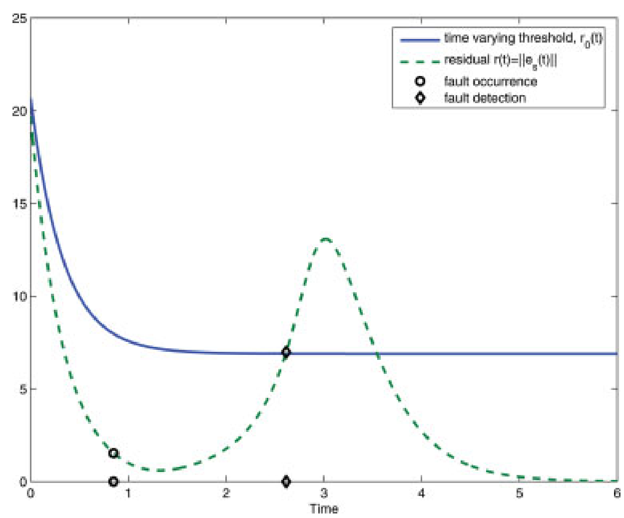

Figure 3. Evolution of residual signal *r*(*t*) and its time-varying threshold r_0(*t*) with incipient fault.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

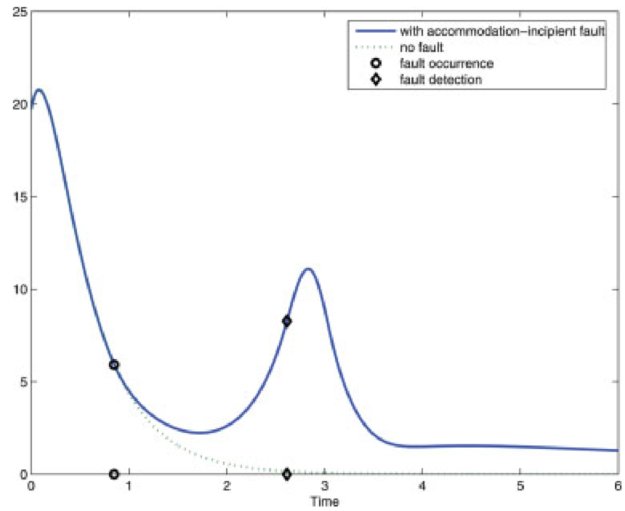

Figure 4. Evolution of state norm with incipient fault.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

one may observe that the parameter estimate does not converge to the true parameter $\theta = 0.59$.

To study the effects of the actuator location on both the detection time and the performance of the fault accommodating controller, we consider the following actuator location optimization, constrained in the set Ψ

$$\psi = \arg \min_{\psi \in \Psi} ||(sI - (\mathcal{A}_s - \mathcal{B}_s(\psi)\mathcal{K}_s(\psi)))\bar{\mathcal{D}}_s||_\infty$$

where $\bar{\mathcal{D}}_s$ denotes the "worst"constant distribution representing the "worst" uncertain term $\widetilde{\Theta}^* g(\bar{x}_s(t))$ in (26). For the LQR-based controller design, the matrices $Q = 0.01I$ and $R = 1$ were used. The incipient fault occurred at the same time $T = 0.85$. The optimal actuator location was chosen as the one that provided the minimum aforementioned norm while for the suboptimal case, still chosen in Ψ, was taken as the admissible locations that yielded the maximum of the norm. Table 1 summarizes the actuator locations for the optimal and suboptimal cases, along with the detection times and delay times for each of the two cases.

Table 1. Optimal and Suboptimal Actuator Locations and their Effects on Detection Time

Case	Actuator location	Detection time t_d	Delay time $\Delta t = t_d - T_c$
Optimal	$(-0.14\pi, 0.34\pi, 0.62\pi)$	1.76	0.91
Suboptimal	$(-0.70\pi, 0.06\pi, 0.38\pi)$	2.99	2.14

Figure 6 depicts the evolution of the residual signal with optimal and suboptimal actuators. The detection time for the optimal case is $t_d = 1.760$, whereas the one for the suboptimal case is $t_d = 2.990$. It can be observed that the choice of the actuator location significantly affects the detection time. Another effect of the actuator location can be observed in Figure 7, where the effects on the system performance are similar to those for fault detection. Since the fault can be detected much earlier with the optimal actuators, the system can have a more effective fault accommodation. The state norm in the case of an optimal actuator converges to zero much faster than the case of suboptimal actuators, and can minimize large deviations of the state from the equilibrium.

Conclusions and Future Work

We have considered a class of nonlinear distributed parameter systems governed by transport-reaction with component faults. The time profile of the fault assumed was general enough to allow both abrupt and incipient faults. Using first an adaptive detection and diagnostic observer, first the time occurrence of the fault was estimated using a time varying threshold and the type of component fault was diagnosed by

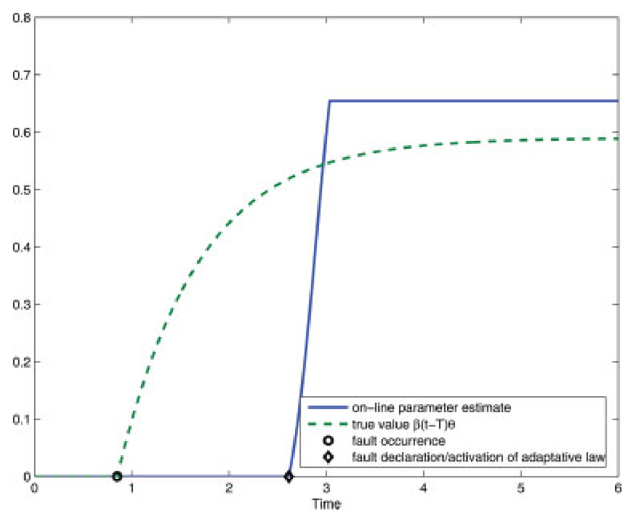

Figure 5. Evolution of incipient fault parameter $\beta(t - T)\theta$ and its adaptive estimate with incipient fault.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

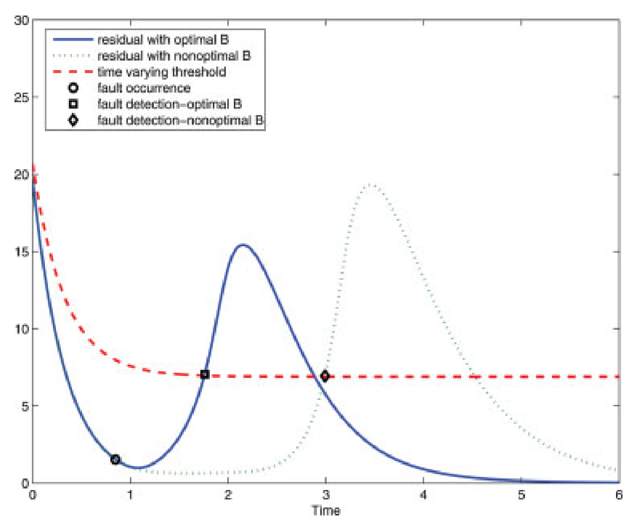

Figure 6. Evolution of residual signal with optimal and suboptimal actuator locations for the case of incipient fault and fault accommodation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

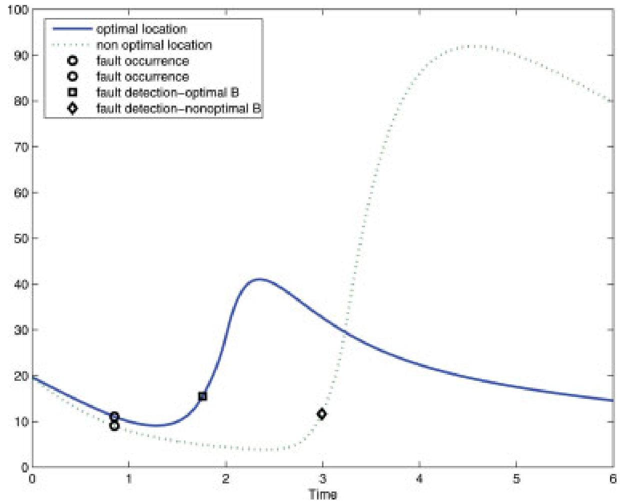

Figure 7. Evolution of state norm with optimal and suboptimal actuator locations for the case of incipient fault with fault accommodation.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

using adaptive estimates via the online approximator of the component fault. The effects of the component fault on the controller performance were addressed via fault accommodation; such an accommodation took the form of control reconfiguration activated at the onset of fault detection. To further enhance robustness an actuator placement scheme was introduced whose goal was to minimize delay time and bring the closed loop performance to the levels of the controller for the healthy system. Extensive simulation studies of the Kuramoto-Sivashinsky equation were included in order to demonstrate the success of the proposed fault detection, diagnosis and accommodation scheme.

An immediate extension that is currently under consideration by the authors is the issue of a fault detection using only partial measurements. Following the enclosed actuator placement scheme, the issue of sensor placement for improved detection and performance enhancement becomes relevant and various optimization schemes for both actuator and sensor placement for enhanced fault tolerability, detection and performance preservation are to be considered.

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Appendix A

Proof of Lemma 1: Prior to the fault, the state observation error is given by

$$\begin{aligned}&\dot{e}_s(t) = (\mathcal{A}_s - \mathcal{L}_s)e_s(t) + \mathcal{N}_s(\bar{x}_s(t)) \qquad 0 \le t \le T\\ &e_s(0) = e_{s0} \neq 0\end{aligned}$$

Hence, its norm bound will be given by the threshold $r_0(t)$. After the fault occurrence, the state observation error is governed by

$$\begin{aligned}&\dot{e}_s(t) = (\mathcal{A}_s - \mathcal{L}_s)e_s(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t-T_c)\Xi_s(\bar{x}_s(t))\\ &e_s(T_c) = e_{sT_c} \neq 0\end{aligned}$$

having a solution

$$\begin{aligned}e_s(t) &= e^{(\mathcal{A}_s-\mathcal{L}_s)(t-T_c)}e_s(T_c)\\ &\quad + \int_{T_c}^{t} e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}(\mathcal{N}_s(\bar{x}_s(\tau)) + \beta_c(\tau-T_c)\Xi_s(\bar{x}_s(\tau)))d\tau\end{aligned}$$

The residual signal $r(t)$, given by the norm of $e_s(t)$, is

$$\begin{aligned}r(t) &= \|e_s(t)\|_\infty\\ &= e^{-\alpha(t-T_c)}\|e_s(T_c)\| + \int_{T_c}^{t} e^{-\alpha(t-\tau)}\|\mathcal{N}(\bar{x}_s(\tau))\|d\tau\\ &\quad + \int_{T_c}^{t} e^{-\alpha(t-\tau)}\beta_c(\tau-T_c)\|\Xi_s(\bar{x}_s(\tau))\|d\tau\\ &\le r_0(t) + \left|\int_{T_c}^{t} e^{-\alpha(t-\tau)}\beta_c(\tau-T_c)\|\Xi_s(\bar{x}_s(\tau))\|d\tau\right|\end{aligned}$$

Hence, a fault is declared when $r(t)$ exceeds $r_0(t)$.

Appendix B

Proof of Lemma 2: The state diagnostic error $e_d(t) = \bar{x}_s(t) - \hat{x}_d(t)$ is governed by

$$\begin{aligned}\dot{e}_d(t) &= (\mathcal{A}_s - \mathcal{L}_d)e_d(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t-T_c)\Xi_s(\bar{x}_s(t))\\ &\quad - \widehat{\Theta}(t)g(\bar{x}_s(t))\\ &= \mathcal{A}_{so}e_d(t) + \mathcal{N}_s(\bar{x}_s(t)) + \beta_c(t-T_c)\Theta g(\bar{x}_s)\\ &\quad - \widehat{\Theta}(t)g(\bar{x}_s(t))\\ &= \mathcal{A}_{so}e_d(t) + \mathcal{N}_s(\bar{x}_s(t)) - [1-\beta_c(t-T_c)]\Theta g(\bar{x}_s(t))\\ &\quad + \Theta g(\bar{x}_s(t)) - \widehat{\Theta}(t)g(\bar{x}_s(t))\\ &= \mathcal{A}_{so}e_d(t) + \mathcal{N}_s(\bar{x}_s(t)) - \Phi(t)\Theta g(\bar{x}_s(t))\\ &\quad - \widetilde{\Theta}(t)g(\bar{x}_s(t)),\end{aligned} \tag{B1}$$

where $\mathcal{A}_{so} \triangleq \mathcal{A}_s - \mathcal{L}_d, \Phi(t) \triangleq 1 - \beta_c(t-T_c)$ and $\widetilde{\Theta}(t) \triangleq \widehat{\Theta}(t) - \Theta$. We consider the following Lyapunov function in order to assess the stability of the diagnostic scheme

$$V = \frac{1}{2}e_d^T\Pi e_d + \frac{1}{2}\widetilde{\Theta}\Gamma^{-1}\widetilde{\Theta} + \frac{1}{2}\mu\frac{\Phi^2}{R} \tag{B2}$$

where Γ is the adaptive gain matrix, $\mu > 0$ is a tuning parameter (weight), and R relates to the rate of change of Φ via $\Phi = -R\Phi$. The parameter Φ is added in (28) for analysis purposes, and it allows one to separate the parameter Θ from its time variation via the fault time profile $\beta_c(t - T)$. The parameter error, used below for the stability analysis, is governed by

$$\dot{\widetilde{\Theta}}(t) = \dot{\widehat{\Theta}}(t) = \mathcal{P}\{\Gamma g(x_s(t))e_d^T(t)\Pi\} \tag{B3}$$

The time derivative of Eq. B2, evaluated along the trajectories of the state diagnostic and parameter error Eqs. B1 and B3, is given by

$$\dot{V} = \frac{1}{2} e_d^T \left(\mathcal{A}_{so}^T \Pi + \Pi \mathcal{A}_{so} \right) e_d + e_d^T \Pi \mathcal{N}_s(\bar{x}_s(t))$$
$$- e_d^T \Pi \Phi(t) \Theta g(\bar{x}_s(t)) - e_d^T \Pi \widetilde{\Theta}(t) g(\bar{x}_s(t)) + \widetilde{\Theta} \Gamma^{-1} \dot{\widetilde{\Theta}} - \mu \Phi^2$$
$$= -\frac{1}{2} e_d^T Q e_d + e_d^T \Pi \mathcal{N}_s(\bar{x}_s(t)) - e_d^T \Pi \Phi(t) \Theta g(\bar{x}_s(t))$$
$$- e_d^T \Pi \widetilde{\Theta}(t) g(\bar{x}_s(t)) + \widetilde{\Theta} \Gamma^{-1} \dot{\widetilde{\Theta}} - \mu \Phi^2 \quad \text{(B4)}$$

where we used the fact that $\mathcal{A}_{so}^T \Pi + \Pi \mathcal{A}_{so} = -Q$ and $\Phi = -R\Phi$. Following a similar analysis in,[47] one uses the fact that the projection modification will make the derivative of the Lyapunov function "more" negative. Using the assumption of the uniform boundedness of $\bar{x}_s$, we have that $\sup_{t \geq T_c} \Xi\ (\bar{x}_s(t)) = \sup_{t \geq T_c} \Theta g(\bar{x}_s(t)) = c_1$. Then we arrive at

$$\dot{V} \leq -\frac{1}{2} \lambda_{\min}(Q) |e_d|^2 + \lambda_{\min}(\Pi) |e_d| |\mathcal{N}_s(\bar{x}_s(t))|$$
$$+ c_1 \|\Pi\|_2 |e_d| |\Phi| - \mu |\Phi|^2 \quad \text{(B5)}$$

Using twice the inequality $2\alpha\beta \leq \alpha^2/\varepsilon + \varepsilon\beta^2$ for some $\varepsilon > 0$ in the previous expression, we then have

$$\dot{V} \leq -\left(\frac{1}{4} \lambda_{\min}(Q) |e_d|^2 + \frac{\mu}{2} |\Phi|^2 \right) + c_1 \|\Pi\|_2 |e_d| |\Phi|$$
$$- \frac{1}{4} \lambda_{\min}(Q) |e_d|^2 - \frac{\mu}{2} |\Phi|^2 + \lambda_{\max}(\Pi) |e_d| |\mathcal{N}_s(\bar{x}_s(t))|$$
$$\leq -\left(\frac{1}{4} \lambda_{\min}(Q) |e_d|^2 + \frac{\mu}{2} |\Phi|^2 \right) + c_2 |\mathcal{N}_s(\bar{x}_s(t))|^2 \quad \text{(B6)}$$

where

$$\mu = \frac{c_1^2 \|\Pi\|_2^2}{\lambda_{\max}(Q)}, \qquad c_2 = \frac{2\lambda_{\max}^2(\Pi)}{\lambda_{\max}(Q)}.$$

When $(\frac{1}{4} \lambda_{\min}(Q) |e_d|^2 + \frac{\mu}{2} |\Phi|^2) > c_2 |\mathcal{N}_s(\bar{x}_s(t))|^2$, we have $\dot{V} \leq 0$. The uniform boundedness assumption of $\mathcal{N}_s(\bar{x}_s(t))$ implies the uniform boundedness of e_d and $\widehat{\Theta}$. Following the analysis in,[47] one can infer that the extended L_2 norm of the state diagnostic error over any finite time interval is, at most, of the same order as the extended L_2 norm of the unmodeled/nonlinear dynamics $\mathcal{N}_s(\bar{x}_s(t))$. In the absence of the unmodeled/nonlinear dynamics, one can easily show, via the application of Barbălat's lemma,[38] that the state diagnostic error converges to zero asymptotically $\lim_{t \to \infty} e_d(t) = 0$. In the latter case, when the additional condition of persistence of excitation is imposed, then parameter convergence in the sense of $\lim_{t \to \infty} \widehat{\Theta}(t) = \Theta$ is guaranteed. Specifically, when nonlinear/unmodeled dynamics $\mathcal{N}(x)$ is assumed zero, then the aforementioned state diagnostic error (B1) along with the online approximator (B3) reduce to

$$\begin{cases} \dot{e}_d(t) = \mathcal{A}_{so} e_d(t) - \Phi(t) \Theta g(\bar{x}_s(t)) - \widetilde{\Theta}(t) g(\bar{x}_s(t)) \\ \dot{\widetilde{\Theta}}(t) = \dot{\widehat{\Theta}}(t) = \mathcal{P}\left\{ \Gamma g(x_s(t)) e_d^T(t) \Pi \right\} \end{cases}$$

The aforementioned is placed in the general form found in,[21] and the proof of parameter convergence then follows from standard arguments of convergence of adaptive parameter estimators.

Appendix C

Following the formulation adopted in,[32,48] we consider the abstract formulation of the aforementioned PDE, under which the system is mathematically represented through an evolution equation in the appropriate Hilbert space. We, thus, consider as an appropriate state space the space of square integrable periodic functions with zero mean, defined via

$$H = \dot{L}^2(\Omega) = \left(\phi \in L^2(\Omega), \int_{-\pi}^{\pi} \phi(\xi) d\xi = 0 \right)$$

with inner product $\langle \cdot, \cdot \rangle$ and corresponding induced norm $|\cdot|$. Associated with the aforementioned, are the two interpolating spaces V and V' given by[49]

$$V = \dot{H}_p^2(\Omega), \qquad V' = H^{-2}(\Omega)$$

A Gelf'and triple space naturally emerges

$$V \to H \to V' \quad \text{(C1)}$$

with both embeddings being dense and continuous.[37,50] Here V is a Sobolev space with norm denoted by $\|\cdot\|_{\boldsymbol{v}}$, and $\boldsymbol{V'}$ denotes the conjugate dual of V (i.e., the space of continuous conjugate linear functionals on V). Let $\|\cdot\|^*$ denote the usual norm on V'. In particular, it is assumed that $|\phi| \leq c_V \|\phi\|_V$ for some positive constant c_V. The notation $\langle \cdot, \cdot \rangle$ will also be used to denote the duality pairing between V' and V induced by the continuous and dense embeddings given in Eq. C1 previously.

Define the operator $A{:}V \to V'$ by

$$\langle A\phi, \psi \rangle = \int_{-\pi}^{\pi} \frac{d^2\phi(\xi)}{d\xi^2} \frac{d^2\psi(\xi)}{d\xi^2} d\xi, \quad \phi, \psi \in V$$

with domain $D(A) = \dot{H}_p^4(\Omega)$. One can easily show that A^{-1} is compact and symmetric operator,[48] and as a consequence, it has a set of orthonormal eigenfunctions that form a basis in $\dot{L}^2(\Omega)$.[51] Furthermore, we define the linear (Laplacian) operator $L : V \to V'$

$$\langle L\phi, \psi \rangle = -\int_{-\pi}^{\pi} \frac{d\phi(\xi)}{d\xi} \frac{d\psi(\xi)}{d\xi} d\xi$$

as well as the bilinear form $\Gamma{:}V \times V \to V'$ via the following expression

$$\langle \Gamma(\phi, \psi), \chi \rangle = \int_{-\pi}^{\pi} \phi(\xi) \frac{d\psi(\xi)}{d\xi} \chi(\xi) d\xi, \quad \phi, \psi, \chi \in V$$

The input operators $B_i(\xi) : \mathbb{R} \to V'$, $i = 1, \ldots, m$ are parameterized as follows

$$\langle B_i(\xi) u_i, \phi \rangle = \int_{-\pi}^{\pi} b_i(\xi) \phi(\xi) d\xi u_i(t), \quad \phi \in V$$

and similarly, the component fault operator below

$$\langle \Xi\phi, \psi \rangle = -\int_{-\pi}^{\pi} \theta \frac{d\phi(\xi)}{d\xi} \frac{d\psi(\xi)}{d\xi} d\xi, \quad \phi \in V$$

Clearly, one may observe that the parameter Θ is in fact a scalar parameter θ, and that the function $g(x_s)$ is basically the Laplacian operator. Within the aforementioned framework, the original spatially distributed dynamics (1) can then be represented as an evolution equation of the following form

$$\frac{d}{dt}x(t) = -vAx(t) - Lx(t) + \Gamma(x(t), x(t)) + \sum_{i=1}^{m} B_i(\xi)u_i(t) + \beta(t-T)\Xi(x(t)), \quad \text{in } V'.$$

By setting $\mathcal{A}x = -vAx - Lx, \mathcal{B}(\Xi) = [B_1(\xi) \cdots B_m(\xi)]$ and $u(t) = [u_1(t) \cdots u_m(t)]^T$ one can rewrite the earlier equation as follows

$$\frac{d}{dt}x(t) = \mathcal{A}x(t) + \Gamma(x(t), x(t)) + \mathcal{B}(\Xi)u(t) + B(t-T)\Xi(x(t)), \quad \text{in V}' \quad \text{(C2)}$$

A finite-dimensional approximation of the evolution system of (Eq. C2), realized through the slow eigenmodes of the differential operator $\mathcal{A}$, can be derived, as it naturally emerges from the time-scale decomposition of the operator's eigenspectrum.[30–32] This can be elucidated from the eigenvalues and eigenfunctions of the operator which are given by $\lambda_j = -v_j^4 + j^2$, $\phi_j(\xi) = \frac{1}{\sqrt{\pi}}\sin(j\xi)$, $j = 1,2,\ldots$. In this study, we adhere to the exposition presented in earlier work by Christofides and co-workers.[35,36,52] Following the analysis summarized in the section the following equivalent form of the process dynamics is obtained

$$\frac{dx_s}{dt} = \mathcal{A}_s x_s + \mathcal{P}_s\Gamma(x,x) + \sum_{i=1}^{m} \mathcal{P}_s B_i(\xi)u_i(t) + \beta(t-T)\mathcal{P}_s\Xi(x(t))$$

$$\frac{dx_f}{dt} = \mathcal{A}_f x_f + \mathcal{P}_f\Gamma(x,x) + \sum_{i=1}^{m} \mathcal{P}_f B_i(\xi)u_i(t) + \beta(t-T)\mathcal{P}_f\Xi(x(t))$$

$$x_s(0) = \mathcal{P}_s x(0), \qquad x_f(0) = \mathcal{P}_f x(0).$$

Assuming that n modes sufficiently capture the slow/unstable process behavior of the system, $\mathcal{A}_s$ is an n-dimensional matrix with diagonal structure $\mathcal{A}_s = \text{diag}\{\lambda_i\}$, where $\{\lambda_i,\ldots,\lambda_n\}$ are the slow eigenvalues associated with H_s, and the unbounded operator $\mathcal{A}_f$ the infinitesimal generator of an exponentially stable C_0 semigroup. Furthermore, under standard assumptions, one can show that

$$\Gamma_s(x,x) = \mathcal{P}_s\Gamma(x,x) = \Gamma_s(x_s + x_f, x_s + x_f)$$

$$\Gamma_f(x,x) = \mathcal{P}_f\Gamma(x,x) = \Gamma_f(x_s + x_f, x_s + x_f),$$

are Lipschitz continuous vector functions.

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